Introduction
The image’s Power Spectrum (PS) is widely used for automated defocus and astigmatism corrections [1], for blind deconvolution procedures [2], [3], and for other purposes.

APEX Method for Blind Deconvolution

![APEX PS approximation profile](image)

The iteratively obtained APEX approximation in principle requires a rotationally symmetric PS (and profiles).

Projection Method

This method also handles a not rotationally symmetric PS, for instance due to the specimen’s geometry or machine aberrations, such as astigmatism (see the second row of Figure 3). The projection method approximates the logarithm of the PS $P(u)$ with a finite linear combination $S(u)$ of analytical shapes $f_i(u)$, i.e.,

$$ S(u) = \sum_i c_i f_i(u), \text{ where } u = [u, v] \text{ - frequency coordinates.} \tag{1} $$

The coefficients $c$ solve $Ac = b$ where

$$ \int_{\Omega} (P(u) - S(u)) f_i(u) du = 0, \forall i \Leftrightarrow \sum_j c_j \int_{\Omega} f_j f_i du = \int_{\Omega} P f_i du, \forall i. \tag{2} $$

The system is solved non-iteratively (uses a direct solver). The Least Squares Difference (LS) between the PS matrix representation $P \in \mathbb{R}^{N \times M}$ and its approximation $S \in \mathbb{R}^{N \times M}$ is

$$ LS := \sqrt{\frac{1}{NM} \sum_{n,m} (P_{nm} - S_{nm})^2} \tag{3} $$

The approximation is considered to be fine if the difference is around the level of the noise. Typical shape functions are the monomials to obtain the moments, or for numerical robustness Legendre polynomials or Chebyshev polynomials.

Numerical Experiment in Matlab

The experimental 442x442 images were obtained with FEI’s Strata SEM. We apply a 20th Chebyshev shape function projection method before the FFTShift operation (see Figure 2). Results of $P$ and related projection approximation $S$ are shown in Figures 2 – 4. Figure 2 is the second column of Figure 3 except for the profiles. In Figure 2 results are before the FFTShift operation. In Figures 3 and 4, they are after. The Figures provide the LS just above their subfigures, it is about 5%. This is not large, taking into account the high amount of noise in $P$. The approximation is smoother than the PS, thus the projection can be used as PS noise reduction.

Setup and solution time

<table>
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<th>Data sets</th>
<th>20&lt;sup&gt;4&lt;/sup&gt; shapes $f_i$</th>
<th>$A$</th>
<th>$b$</th>
<th>$c$</th>
</tr>
</thead>
<tbody>
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<td>Time (sec)</td>
<td>4.187</td>
<td>599 826</td>
<td>2069</td>
<td>0</td>
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</tbody>
</table>

The setup of $A$ and $f$ takes the most time. However, they can be computed in advance because they do not depend on the PS. The calculation of $c$ for a specific PS consists of the time to set up $b$ and to solve for $c$, i.e., around 2 seconds.

References

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